Synchronous amplification factor, Q_S

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In the course of designing or defining a dynamic system, it is often desirable to quantify the damping in order to predict the forced vibration response at a resonance. The purpose of this article is to clarify the ambiguity associated with a commonly used term, Q_S, the synchronous amplification factor, and to discuss some aspects of quantifying resonant responses of rotating machinery.

The amplification factor is used in vibration theory as a measure of system susceptibility to periodic forced excitation at resonances. Attempts have been made to use the synchronous amplification factor as a measure of the stability margin against fluid-induced instability, but far better measurements are now available for the purpose [1,2,3,5,6,7]. The term synchronous, as in "synchronous amplification factor" or "synchronously perturbed," is used in relation to rotor lateral response. It indicates the forcing frequency (ω) is equal to the shaft speed (Ω), as is commonly found in the form of rotor unbalance. This relation allows for a fair amount of mathematical simplification. The term "synchronous" is also a flag that should alert one to the fact that any data used should be 1X filtered to the shaft speed of the rotor in order to eliminate noise from responses at other frequencies.

The synchronous amplification factor, Q_S, of the isotropic rotor synchronous 1X response can be defined four separate ways:

$$Q_S = \frac{A_{RES}}{A_0} \quad \text{for F = constant (peak ratio method)} \qquad (1)$$

$$Q_S = \frac{A_{RES}}{A_{(\Omega=\infty)}} \quad \text{for } F = m_u r_u \Omega^2 \text{ (peak ratio method)} \qquad (2)$$

where:

F = exciting force magnitude

m_u, r_u = mass and radius of unbalance, respectively

Ω = rotative speed

$$Q_S = \frac{\Omega_{RES}}{\Omega_2 - \Omega_1} \quad \text{(half power bandwidth method)} \qquad (3)$$

$$Q_S = \left[\frac{\Delta\phi}{\Delta\Omega}\right]\frac{\Omega_{RES}}{2} \quad \text{(phase slope method)} \qquad (4)$$

Note: *units must be radians for angular measure and radians per second for angular velocity.*

Note that these formulas apply only to rotors with widely spaced modes. For instance, anisotropic rotors or double overhung rotors have closely spaced modes and are not covered by these equations.

In Eq. (1), Q_S is defined as the ratio of the resonant peak response amplitude (A_{RES}) to the nonresonant response amplitude (A_0). A_0 is defined as the response amplitude at very low frequency. This method is appropriate if the excitation force magnitude is constant. In Eq. (2), Q_S is defined as the ratio of the resonant peak response amplitude (A_{RES}) to the nonresonant response amplitude at high speed ($A_{\Omega=\infty}$). This method is appropriate if the excitation force is proportional to frequency squared, as in the rotating unbalance excitation. $A_{\Omega=\infty}$ is defined as the lowest response amplitude at rotative speed above the resonant mode of interest, but below the frequency when the next mode starts being active. Judgment is required when selecting the value of $A_{\Omega=\infty}$. For example, if the amplitude is zero at some point, then Q_S becomes infinite using this definition. In cases like this, it is best to use one of the other definitions of Q_S. Eq. (3) is the half power bandwidth method definition of Q_S. In the numerator, Ω_{RES} is defined as the rotative speed at which the peak resonance amplitude occurs. The denomina-

tor contains the difference of Ω_2 and Ω_1 defined as follows: $\Omega_1 < \Omega_{RES}$ and $\Omega_2 > \Omega_{RES}$ are the rotative speeds where response amplitudes are lower than resonance amplitudes by 3dB or are equal to $(A_{RES})/(\sqrt{2})$

Eq. (4) defines Q_S in terms of the slope of the response phase at resonance. $\Delta\phi$ is defined as the difference between two response phase values close to the resonance, one measured at each side of the phase at resonance. $\Delta\Omega$ is defined as the difference between the rotative speeds corresponding to the points used in calculating $\Delta\phi$. The ratio $\Delta\phi/\Delta\Omega$ must be expressed as a positive number. See Figure 1 for a graphical representation of this method. The data used for calculating this difference should bracket the resonance point closely to get as close to an instantaneous differential as possible. Ω_{RES} is, as in the other definitions, the rotative speed at resonance.

In fluid (liquid or gas) handling machines and machines with liquid or gas bearings, seals, or other rotor to stator interfaces, the damping (D) is modified by the term (1-λ) to become the effective damping, D(1-λ). Lambda (λ) is the fluid circumferential average velocity ratio for fluid interacting with the rotor and is a measure of the circumferential flow strength. D by itself is always a positive number. Damping in a bearing or seal may be very small, seemingly approaching zero, if length is short, clearance is high with low eccentricity position of the journal, or if the fluid viscosity is low, but damping is ***never*** negative, in spite of the multitude of books and papers that use the expression "negative damping." What actually becomes negative is the quadrature stiffness [3]. However, depending on λ, the effective damping can be greatly modified. Typically for fluid bearings, $0.3 < \lambda < 0.48$; for seals without antiswirl, $0.4 < \lambda < 0.5$; and for some fluid handling machines with forward preswirling or significant recirculation flow, λ may even be far greater than 1. If antiswirl fluid injection is used, λ is reduced, resulting in an increased threshold of stability. It is theoretically possible to reduce λ to the point of being negative, but a negative λ is very difficult to achieve in reality. For simple rotor systems with rolling element bearings, λ may be assumed to be equal to zero.

Fluid involved in circumferential motion due to shaft motion contributes in two ways to the dynamics of the rotor system. One term is the radial damping force, $+j\Omega Dr$, which can be looked at as an interaction where the shaft pushes on the fluid (r is the rotor lateral displacement). Conversely, there is the tangential wedge force term, $-j\Omega D\lambda r$, which can be viewed as the fluid pushing back on the shaft. These forces have opposite direction, thus the quadrature stiffness is the difference between the effects. By adding the terms and factoring, it can be seen that the magnitude of their opposition is directly proportional to (1-λ).

Fluid involved in circumferential motion has a strong effect on the value of Q_S, since it changes the damping from D to D(1-λ) for a synchronously perturbed (unbalanced) system. Since λ is often just below one half, then the observed "damping" is correspondingly about one half of the actual damping at resonance speed. If λ is between 0 and 1, Q_S is higher indicating lower "effective damping."

Figure 2 shows Eqs. (2) & (3) applied to rotor 1X response data reduced in Bode and polar plot format. These equations apply to all modes of a system. If a machine passes through several modes during startup/shutdown, these relations can be applied to each mode of interest.

Another important feature of λ is that it provides a tangential force coupling of the modes in the vertical and horizontal axes (or any two arbitrary orthogonal axes mutually perpendicular to the rotor shaft, for that matter). The equation of motion for a one-complex-degree-of-freedom, synchronously perturbed (unbalanced), isotropic rotor system with fluid interaction is as follows:

$$M\ddot{r} + D\dot{r} + Kr - jD\lambda\Omega r = Fe^{j(\Omega t+\delta)} \quad (5)$$

where: $r = x + jy$, $j = \sqrt{-1}$

like rectangular to polar conversion:

$$x = |r|\cos\theta, y = |r|\sin\theta,$$

$$|r| = \sqrt{x^2 + y^2}, \theta = \arctan\frac{y}{x}$$

• = d/dt

M = modal mass

D = damping

K = modal stiffness

F = external force magnitude

δ = (delta) the angular orientation of F

Note: The Euler expression

$$e^{j(\Omega t+\alpha)} = \cos(\Omega t + \alpha) + j\sin(\Omega t + \alpha) \quad (6)$$

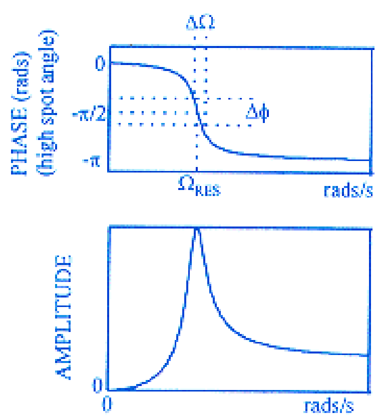

Figure 1
Phase slope definition of Qs

The rotor synchronous response is:

$$r = Ae^{j(\Omega t+\alpha)} = A\cos(\Omega t+\alpha) + jA\sin(\Omega t+\alpha) \quad (7)$$

where:

$$A = \frac{F}{\sqrt{(K - M\Omega^2)^2 + \Omega^2 D^2 (1-\lambda)^2}} \quad (8)$$

$$\alpha = \delta + \arctan\frac{-\Omega D(1-\lambda)}{K - M\Omega^2} \quad (9)$$

An understanding of this fluid-related tangential force is critical to gaining an insight into the behavior of fluid handling machines. The magnitude of the tangential force is directly proportional to shaft speed, lambda, and damping values. The references listed at the end of this article provide further discussion of the effects of lambda and the fundamental dynamics of fluid bearing rotor systems. Copies are available by contacting the editor of this magazine. The address, fax and phone numbers of the Orbit magazine are listed on the table of contents page.

For an isotropic rotor with only one mode and low damping, Q_S is related to the damping factor ζ (zeta) by the following relationship:

$$Q_s = \frac{1}{2\zeta(1-\lambda)} \quad (10)$$

where ζ is the damping ratio defined as the ratio of current damping (D) to critical damping (D_{CR}), defined as follows:

$$\zeta = \frac{D}{D_{cr}} \quad (11)$$

$$D_{cr} = 2\sqrt{KM} \quad (12)$$

In conclusion, this article has discussed Q_S and a special case of phase slope at resonance for a one lateral mode rotor. It must be restated that the relations presented here are only true for isotropic synchronously perturbed rotors. As stated earlier, these formulas apply only to rotors with widely spaced modes. For instance, anisotropic rotors or double overhung rotors have closely spaced modes and are not covered by these equations. Rotating machines with mismatched radial stiffness ***must*** be treated by more complex rules. Furthermore, the behavior becomes quite different when nonsynchronous perturbation is introduced. There will be follow-up articles in Orbit that will discuss these more complicated cases.

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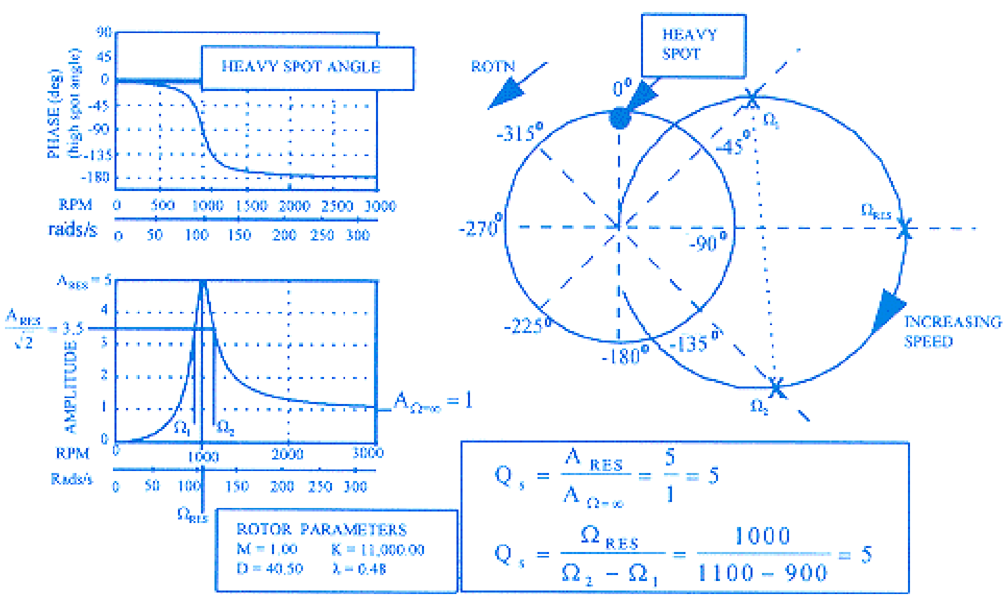

Figure 2

Equations 2 & 3 applied to 1X response data and reduced in Bode and polar plot formats.